Numerical and Experimental Studies on Thermal Contact Resistance at Solid-Solid Interfaces

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In this paper, numerical simulations and measurements of thermal contact conductance (TCC) at the interface of two cylinders in contact are carried out. The random model of surface roughness is developed and the non-dimensional basic equations are solved based on a gird system with equiperipheral interval in azimuth direction that can express reasonably the real contact spot distribution. The effects of the contact pressure, thermal conductivity of interstitial medium and mean absolute slope on the TCC were clarified by using a network method. In the experiments, four pairs of brass cylinders each of which has similar surface topology are used for the TCC measurements. A hysteresis nature of TCC versus contact pressure was observed in the first loading cycle. The present numerical results shows that the TCC increases linearly with the mean absolute slope of the surfaces even at the same mean roughness. Such a tendency agrees well with those measured.

KEY WORDS: thermal contact conductance; thermal contact resistance; equiperipheral grid; numerical simulation; experiment.

1. INTRODUCTION

The thermal contact resistance (TCR) or thermal contact conductance (TCC=1/TCR) at the interface of two solids in contact plays an important role in many engineering applications such as the cooling of electronic devices [1]. Although a large number of studies [2-8] since the 1950s have been carried out to clarify the effects of the surface topography, that is, surface roughness and mean surface absolute slope on TCR, the existing analytical models are applicable only to the limiting cases [9] and it is still difficult to establish a general expression that can predict accurately the TCR in practical engineering applications. The reasons are mainly due to the difficulties to characterize the actual surface topology quantitatively and accurately. The measurement of the mean surface slope, for example, largely depends on the resolution of the roughness measurement instrument. To overcome these difficulties, Tomimura et. al. [10] and Zhang et. al. [11] developed a new surface roughness model based on the superposition of sine waves with random parameters. The roughness model has been proved to be valid and can be used effectively in the numerical simulations through comparison with the corresponding experimental results.

In this paper, detailed numerical simulations of thermal contact conductance were carried out for various surface configurations and contact pressures, where the basic equations and boundary conditions were non-dimensionalized based on the mean surface roughness. A gird
system with equiperipheral interval in azimuth direction was developed to express reasonably
the real contact spot distribution and a network method based on this grid system was used to
calculate the TCC. The effects of the contact pressure, thermal conductivity of interstitial
medium and mean absolute slope on TCC were investigated. Further, the TCC of the four
pairs of brass cylinders in contact was measured. The actual surface roughness of all test
cylinders was measured and analyzed and the four combinations of test cylinders with similar
surface topology were chosen. A hysteresis nature of pressure on the TCC was observed in the
first loading cycle. The present numerical results agree well with experimental results.

2. NUMERICAL SIMULATIONS

2.1 Physical Model in Simulations

Figure 1 shows the physical model and coordinates system for the simulations of heat
conduction through the interfaces. A pair of specimens of length \( L \) and diameter \( D \) is pressed
together with an equivalent mean contact pressure \( p_m \). Specimens I and II have the thermal
conductivities \( k_I \) and \( k_{II} \) and maximum roughness heights \( R_{maxI} \) and \( R_{maxII} \), respectively. A
uniform heat flux, \( q_m \), is assumed at the bottom surface of the lower specimen (\( z=0 \)), a uniform
temperature, \( T_c \), is assumed at the top surface of the upper specimen (\( z=2l \)), and the side
surface of the two specimens (\( r=D/2 \)) is thermally insulated.

![Fig. 1. Physical model for simulations of heat conduction through the interfaces.](image)

2.2 Non-Dimensional Surface Roughness Model

The present surface roughness model in dimensional form was already described in
Ref. [11]. A typical rough surface constructed with some average roughness and minimum
and maximum wavelengths is shown in Fig. 2. This model is confirmed to have a self-affinity
and exhibit a height distribution close to the ‘normal’ or Gaussian probability distribution.

In this paper the model is non-dimensionalized based on the average surface
roughness, \( Ra \), as follows.

\[
Z(R, \theta) = B \sum_{i=1}^{n} \sin \left[ \frac{2 \pi R \left( \cos \left( \theta + \alpha_i \right) \right)}{L_i} - \varphi_i \right]
\]

(1)
Fig. 2. Simulated result of surface roughness in cylindrical coordinates using random numbers model \((Ra=2.2\,\mu m, l_{\text{min}}=2\times10^{-4}\,\text{m}, l_{\text{max}}=2\times10^{-2}\,\text{m})\).

\[
\frac{R}{Ra} = \frac{l_i}{Ra}, \quad B = \frac{1}{\pi R_0^2} \int_0^{2\pi} \int_0^{2\pi} \left[ \sum_{i=1}^n \frac{2\pi R \cos(\theta + \alpha_i)}{L_i} - \varphi_i \right] \frac{R}{dR} dR d\theta
\]

Where, \(r\) and \(R\) are the dimensional and non-dimensional radius, \(\theta\) is the angle, \(R_0\) is the dimensionless outer radius of the specimen, \(n\) is the number of superposed sine waves used to construct a rough surface, \(l_i\) and \(L_i\) are the dimensional and non-dimensional wavelengths, \(\varphi_i\) and \(\alpha_i\) are the initial phase and orientation of \(i\)th waves. The parameters \(L_i, \varphi_i\) and \(\alpha_i\) are expressed randomly by the following equations.

\[
L_i = (L_{\text{max}} - L_{\text{min}}) \cdot \text{RND}_{i,1} + L_{\text{min}}
\]

\[
\varphi_i = 2\pi \cdot \text{RND}_{i,2}
\]

\[
\alpha_i = 2\pi \cdot \text{RND}_{i,3}
\]

Here \(\text{RND}_{i,1}, \text{RND}_{i,2}\) and \(\text{RND}_{i,3}\) are the random numbers and \(L_{\text{min}}\) and \(L_{\text{max}}\) are the minimum and maximum dimensionless wavelengths respectively. This model can reproduce the actual surface roughness having an arbitral height distribution and mean slope of roughness by choosing the dimensionless parameters \(L_{\text{max}}, L_{\text{min}}, \alpha_i\) and \(\varphi_i\). It is noted that the mean surface roughness does not appear explicitly.

To analyze the effects of various parameters on the TCC, dimensionless temperature, \(\Theta\), pressure, \(P\), the thermal conductivity ratio of medium-to-specimen, \(K\), and the average absolute slope of rough surface, \(\tan \theta\), are defined as follows.

\[
\Theta = \frac{T - T_c}{q_m Ra / k}, \quad P = \frac{P_m}{\sigma_y}, \quad K = \frac{k}{k_s}, \quad \tan \theta = \frac{1}{N} \sum_{i=1}^{N} |Z_{i+1} - Z_i| / \Delta_i
\]

Here, \(T_c\) is the temperature at the top surface of specimen I, \(q_m\) is the heat flux given at the
bottom surface of specimen II, \( k_s \) is the thermal conductivity of specimen, \( \sigma_y \) is the yield stress, \( N \) is the total number of grids and \( \Delta_i \) is the horizontal interval of two grids. Dimensionless TCC can be defined as follows.

\[
H_m = \frac{h_m Ra}{k} = (\Theta_{1,\text{Interface}} - \Theta_{II,\text{Interface}})^{-1}
\]

(5)

Here, \( h_m \) is the thermal contact conductance, and is defined as the ratio of the heat flux \( q_m \) to temperature drop at the interface \( (T_{1,\text{Interface}} - T_{II,\text{Interface}}) \):

\[
h_m = \frac{q_m}{T_{1,\text{Interface}} - T_{II,\text{Interface}}}
\]

(6)

2.3 Computational Method

We used a network method to solve a three-dimensional heat conduction problem where an equiperipheral grid has been developed for the cylindrical coordinates system. The grid system can express reasonable contact spot distributions at the solid-solid interface. We assumed that the deformation of each asperity is fully plastic, and the change of volume due to the deformation is to be neglected. The Successive Over-Relaxation (SOR) method is used and the iteration is terminated when the maximum temperature difference between the successive steps becomes less than \( 10^{-6} \). Further details of numerical method can be seen in Ref. [11].

2.4 Numerical Results

Figure 3 shows the relationship between the dimensionless TCC and mean pressure at \( \tan \theta = 0.0227 \) with \( K \) as the parameter, where \( K=0 \) corresponds to vacuum and \( K=2.12 \times 10^{-4} \) to combination of air and brass cylinder. In the figure, the experimental results are plotted with symbols □, that is obtained under atmospheric pressure air and brass (\( Ra=2.2 \mu m \)) combination [10]. The numerical result with equiperipheral grid system shown by solid line agrees very well with those corresponding experiments. On the other hand, the solution with the conventional equiangular grid system shows lower values of TCC than the experiments. As shown in this figure, the TCC increases with a power of the mean pressure for all five thermal conductivity ratios. Further the TCC increases with an increase in \( K \), because more heat flux passes through the interstitial medium with higher thermal conductivity of the medium. This reduces the effect of contact pressure, therefore, the power of the mean pressure decreases with an increase in \( K \).

Fig. 4 shows the relationship between the TCC and the average absolute slope of rough surface at \( P=0.0186 \) and in a vacuum condition \( K=0 \). Where the range of abscissa \( \tan \theta \) is widen beyond the normal range encountered in the case of microscale rough surfaces by considering the measurements of Yan and Komvopoulos [7]. They reported the surface roughness profile of a carbon-coated magnetic (rigid) hard disk measured with an atomic
Fig. 3. Relation between contact pressure and TCC (\(\tan \theta = 0.0227\)).

Fig. 4. Relation between average slope (\(\tan \theta\)) and TCC in vacuum.

Fig. 5. Relation between average absolute slope and density of contact spots (\(P = 0.0186\))
Contact spot distributions \( P = 1.86 \times 10^{-2} \), (a) \( \tan \theta = 0.02 \); (b) \( \tan \theta = 0.05 \).

Analyzing the power spectrum density profile of their roughness, such a nanoscale surface roughness is confirmed to have around ten times higher value of mean absolute slope than that of ordinal microscale roughness for a typical ground metal surface. The figure shows clearly that the TCC increases linearly with an increase in \( \tan \theta \). This indicates that the TCR can be greatly reduced when the surfaces with higher mean slope but the same roughness height are in contact for the random rough surfaces. This is attributed to the fact that the density of the contact spot, \( \rho_c \), is greatly increased with the mean slope of roughness as shown in Figure 5. In the figure, the contact pressure is kept constant at \( P = 0.0186 \), therefore, the total true contact area is not changed under the assumption of plastic deformation of asperities. The TCR is caused by the heat flow constriction at the real contact spots. Therefore, if there are more contact spots on the surface, the constriction of heat flow will be reduced and at the same time the TCR.

Figures 6a and 6b show the contact spot distributions at \( \tan \theta = 0.02 \) and 0.05, respectively. Both of them are obtained under the conditions of the same apparent contact area, roughness height and same contact pressure. Thus the total true contact area of two cases must be the same. The larger numbers of contact spots and more uniform distribution is observed in Fig. 6b than in Fig. 6a. The temperature contours at the upper or lower contact surfaces corresponding to the above conditions are shown in Fig. 7, and isotherm lines at the cross-section near the interface and the temperature profiles across the interface are shown in Fig. 8 and 9, respectively. Due to the larger numbers of contact spots and reduced heat flow constriction for \( \tan \theta = 0.05 \), the temperature drop at the interface is smaller than that for \( \tan \theta = 0.02 \). Therefore, the TCC for \( \tan \theta = 0.05 \) becomes higher than that for \( \tan \theta = 0.02 \).
Fig. 7. Temperature distributions at the interfaces 
\(P=1.86 \times 10^{-2}\), (a) \(\tan \theta = 0.02\); (b) \(\tan \theta = 0.05\).

Fig. 8. Temperature distributions at cross sections 
\(P=1.86 \times 10^{-2}\), (a) \(\tan \theta = 0.02\) (A-A); (b) \(\tan \theta = 0.05\) (B-B).
3. TCC MEASUREMENTS

3.1 Experimental Setup

Figure 10 shows the experimental setup for the present measurements. A pair of brass specimens is set on a table. The heater, 5, attached to the bottom of specimen I has the same diameter as the specimens. The cooling block, 8, is mounted on the upper of transducer block. Ten thermocouples are inserted into the specimens along the longitudinal direction to measure the temperature distribution. The outside of specimens is wrapped with the thermal insulator to reduce the heat loss. A balance machine is used to load onto the specimens through the copper block, 7. This can correctly control the load on specimen by changing the weight without the effects of thermal expansion of specimens. The ultrasonic transducer, 3, is fixed at the copper block, but it is not used in the present experiments.

A uniform heat flux was supplied at the bottom of specimen I by supplying an electrical current to the heater. A uniform and constant temperature was kept at the top surface of specimen II by the cooling water. The temperatures of the specimens and the heat rate of the heater were measured by a measurement system including a scanner, two voltmeters and a personal computer. To perform an accurate measurement, the temperature drop at the interface was kept to be larger than 2 K, where the corresponding heat rate was about 30.0 watts. The measurements were done at the steady state condition, when the maximum temperature change over 12 minutes was confirmed to be less than 0.1 K.
3.2 Test Samples and Their Surface Topology

Four pairs of surface-sandblasted brass cylinders were used to measure the TCC. All the cylindrical specimens have 40 mm in diameter and 45 mm in length. Five holes of 0.6 mm in diameter and 6 mm in depth were drilled at each specimen to mount the thermocouples. The interval of neighboring holes is 5 mm and the nearest one is 5 mm away from the contact interface. The roughness of the brass surfaces was measured with the contact stylus profiler (SE-40c, Kozaka Lab). This instrument has a diamond stylus of 5 \( \mu \text{m} \) in radius and the relative error of longitudinal magnification is within 3%. The maximum scanning length is 30 mm that yields 8000 evenly spaced data points. The range of the height measurement is 0.001 -100 \( \mu \text{m} \).

Two arbitrary directions of each surface were measured. Figure 11 shows the measured results of four pairs of surface roughness profiles. It is easily found that those surfaces are quite flat although the average surface roughness and average absolute slope indicated in the captions are different. Pairs \( a \), \( b \) and \( c \) are blasted by the glass beads, and pair \( d \) is by the grains of sand. Although the sizes of the glass beads used for pairs \( a \) and \( b \) are the same, because the processing time of pair \( b \) is longer than that of pair \( a \), the \( Ra \) and \( \tan \theta \) of pair \( b \) are larger than those of pair \( a \). Comparing pair \( c \) with pair \( d \), both of them have the same roughness \((Ra)\), but the average absolute slope of pair \( d \) is 80% larger than that of pair \( c \) because the shape and material of grains are different.
We confirmed through a statistical analysis of the surface roughness height distributions that all of these four pairs satisfy Gaussian distribution of roughness height. Further, from an analysis of the power spectral density (PSD) of the surface roughness, it is proved that the pair d should have the larger average absolute slope than the pair c.

3.3 Data Processing

The temperatures at the five points of each specimen are measured and plotted in Fig. 12, as an example. Because the measurement locations are not very close to the interfaces, the temperatures at each z-plane are almost uniform. Therefore, the measured temperature can be regarded as the plane-averaged local temperature at each position. Since the temperature distribution along the z-direction is linear, the temperature gap ($\Delta T=T_I-T_{II}$) at the interfaces can be obtained from the extrapolation. On the other hand, the heat rate ($Q$) flow across the interface can be calculated from the heat rate generated by the heater. Therefore, we can obtain experimentally the TCC in dimensional and non-dimensional forms by Eqs. (6) and (5), respectively.
3.4 Uncertainty Analysis

The uncertainty of the TCC measurement is considered as follows. The errors caused by the heating current, $\delta I / I$, and voltage, $\delta V / V$, are considered to be $1.4 \times 10^{-4}$ and $7.6 \times 10^{-5}$, respectively. The heat loss caused by both the bottom plate and the side thermal insulator, $\delta q_l / q_l$, is about $3.7 \times 10^{-3}$. The error caused by the diameter measurement, $\delta D / D$, is about $6.7 \times 10^{-4}$. The thermocouples inside the specimen may be located at about $\pm 0.3$ mm away from the ideal spot, this difference results in a maximum error of $6.0 \times 10^{-2}$ in the temperature gradient. The maximum error of temperature measurement, $\delta T / T$, is $2.1 \times 10^{-2}$. Based on the law of error propagation

$$e_t = \sqrt{\left(\frac{\delta I}{I}\right)^2 + \left(\frac{\delta V}{V}\right)^2 + \left(2 \frac{\delta D}{D}\right)^2 + \left(\frac{\delta q_l}{q_l}\right)^2 + \left(\frac{\delta \Delta l}{\Delta l}\right)^2 + \left(\frac{\delta T}{T}\right)^2} \quad (7)$$

The total uncertainty of the present measurements is estimated to be $\pm 6.4\%$.

4. RESULTS AND DISCUSSION

Figure 13 shows the measured results of TCC obtained from the successive loading and unloading. In the figure, pairs $a$, $b$ and $c$ are the results of three complete loading cycles, and pair $d$ is the results of two complete loading cycles. Also is drawn by solid line the present numerical results corresponding to each pair of contact surface. The numerical results are found to be in good agreement with those measured. In all of the figures the first loading cycle has a different path. It means that most of the plastic deformation of asperities occurs in
the first loading cycle. For the first unloading path, the asperities have already been flattened plastically after the first loading to the highest pressure so that the true contact area and TCC increase relatively at the corresponding load. Since the load was not removed completely from the specimens at the beginning of the second and third cycles, the relative position of each specimen was not changed. However, a small amount of plastic deformation is still observed from the hysterics exhibited by the cycles. Although pair \( c \) and pair \( d \) have the same surface roughness \((Ra=2.21 \, \mu m)\), because they have different \( \tan \theta \) \((c: \tan \theta=0.137, \ d: \tan \theta=0.249)\), the measured TCC of pair \( d \) is larger than that of pair \( c \). This result clearly confirms the numerical prediction that the TCC increases with an increase in \( \tan \theta \) for the same mean surface roughness. To make the effect of average absolute slope of surface roughness clear, the relation between average absolute slope, \( \tan \theta \), and dimensionless TCC, \( H_m \), at a constant pressure \((P=0.0057, \ p=1.68 \text{ MPa})\) is shown in Fig. 14. The TCC, \( H_m \), is proportional to the average absolute slope, \( \tan \theta \). Also the present experimental results agree quite well with the numerical ones, although the latter shows a little bit higher than the former.

Fig. 13. Relation between contact pressure and TCC.
5. CONCLUSIONS

Numerical simulations and experiments of TCC at the interfaces of two cylinders in contact have been carried out for various surface configurations and contact pressures. Numerical results show that the dimensionless TCC increases with an increase in the mean pressure, in the thermal conductivity ratio of medium-to-specimen, and in the average absolute slope of rough surface. A hysteresis nature of the contact pressure on TCC was observed experimentally between the first and second loading cycles. The experimental results have proved the numerical prediction that the dimensionless TCC increases with an increase in the mean absolute slope of surface roughness at a constant contact pressure and average surface roughness. The present results show a clue of new way to reduce the thermal contact resistance in the practical engineering.

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