STUDY OF THERMAL PROPERTIES OF POROUS MATERIALS

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Abstract
Thermal properties of materials are examined in this paper. The transient pulse method is used for specific heat, thermal diffusivity and thermal conductivity determination. A new method is applied to the evaluation of ascertained data. This method derives from general relations that were designed for the study of physical properties of fractal structures. The dependence of fractal structures’ (characterized by the fractal dimension $D$ in $E$-dimension space) temperature on the time $t$ is determined using the theory of the space-time fractal field for the different heat power energy and for different pulse width. It is possible to calculate the coefficient $f_a$ (fractal dimension $D$ respectively) for every point of the experimental dependence. The value of this coefficient could be also affected by the geometry of sample or by the finite pulse width. By fitting of these relations to the ascertained data, the fractal dimension of the used heat sources and the given thermal quantities, which are in accordance with the values calculated using standard relations characterizing dynamic thermal field of the specimen, are identified.

Key words: fractal structure, specific heat, thermal diffusivity, thermal conductivity, transient pulse method
1. Introduction

The article deals with the use of new data evaluation method, which was described in [1]. The method results from generalized relations that were designed for study of physical properties of fractal structures [2], [3]. As it is shown these relations are in good agreement with the equations used for the description of time responses of temperature for the pulse input of supplied heat [4], [5], [6]. Comparable outcomes of thermal parameters (specific heat, thermal diffusivity and thermal conductivity) with classical method were obtained.

2. Theory

The dependence of fractal structures’ (characterized by the fractal dimension $D$ in $E$-dimension space) temperature on the distance from heat source $h_T$ and on the time $t$ was determined in [1] using the theory of the space-time fractal field [2], [3]

$$T = \frac{Q}{c_p \rho \left( \frac{8\pi a_0 t}{D - E + 2} \right)^{(D-E)/2}} \left( 1 - \frac{h^2}{4a_0 t} \right)^{(D-E+2)/2}.$$

In this term, $Q$ is the heat supply from the heat source, $c_p$ is the specific heat capacity at constant pressure, $\rho$ is the mass density and $a_0$ is the minimum value of the thermal diffusivity $a = 2a_0/(D - E + 2)$ for fractal dimension equal to the topological dimension of the space ($D = E$).

If the heat diffuses by the significantly smaller speed ($h^2 << a_0 t$, small distances or long times) the terms in parenthesis can be considered as significant in the expansion of exponential function ($1 - x = e^{-x}$) and thus we can write

$$T = \frac{Q}{c_p \rho (4\pi a_0 t)^{(E-D)/2}} \cdot \exp \left( -\frac{h^2}{4a_0 t} \right),$$

where $Q$ is the total heat transferred to the body from the heat source with the thermal conductivity $\lambda = c_p \rho \ a$. The relation (2) is applicable for fractal dimensions $D = 0, 1, 2$ and
topological dimension $E = 3$ published in [4], [5], [6], see Fig. 1.

Fig. 1 Heat flow geometry for a) plane-parallel, b) cylindrical and c) spherical coordinates Euclidean space.

The maximum position can be determined by the derivation of (2) with the time

$$\frac{\partial \log T}{\partial \log t} = \left( \frac{D - E}{2} + \frac{h^2}{4at} \right) = 0.$$  \hfill (3)

From this equation the thermal diffusivity at the time of dependence’s maximum can be determined

$$a = \frac{h^2}{2t_m f_a} = \frac{h^2}{2(E - D)t_m},$$  \hfill (4)

where $f_a$ is a coefficient that characterizes the deformation of the thermal field [6]. This coefficient is equal to one for the ideal plane source ($E = 3$, $D = 2$). The maximum temperature of the response for Dirac thermal pulse is obtained by introducing of the thermal diffusivity (4) in the term (2)

$$T_m = \frac{Q}{c_p \rho} \exp \left( \frac{D - E}{2} \right) \left( \frac{E - D}{2\pi h^2} \right)^{(E-D)/2}.$$  \hfill (5)

From the ratio of equations (6) and (2) and with the use of the term (4)

$$\frac{T_m}{T} = \left[ \frac{t}{t_m} \exp \left( \frac{t_m}{t} - 1 \right) \right]^{\frac{E-D}{2}},$$  \hfill (6)
it is possible to define the coefficient $f_a$ (fractal dimension $D$ respectively) for every point of
the experimental dependence

$$f_a = E - D = \frac{2 \ln(T_m/T)}{\ln(t/t_m) + (t_m/t - 1)}.$$  \hspace{1cm} (7)

The value of the coefficient $f_a$ could be also affected by the geometry of sample [6] or by the
finite pulse width [7].

From term (5) the thermal capacity

$$c_a = \frac{Q}{\rho T_m h_T \sqrt{2\pi \exp(1)}} = \frac{Q}{\rho T_m h^{E-D}} \left( \frac{E-D}{2\pi \exp(1)} \right)^{(E-D)/2}$$ \hspace{1cm} (8)

and thermal conductivity of the studied fractal structure

$$\lambda = c_p \rho a = \frac{Q}{2(E-D)T_m t_m h^{E-D-2}} \left( \frac{E-D}{2\pi \exp(1)} \right)^{(E-D)/2},$$ \hspace{1cm} (9)

where $f_a$ and $f_c$ are the coefficients that characterize the deformation of the thermal field [6],
can be obtained.

The (6) represents time-temperature dependencies (according equation (2)) calculated for
spherical ($D = 0$), cylindrical ($D = 1$), planar ($D = 2$), and cubic ($D = 3$) geometry of the heat
source (see Fig. 1). It is evident from the (6) and from the equation (3) that for $D = E$ the
function meets maximum for the time $t \to \infty$.

All dependences for the long time intervals converge to the asymptote, which is longitudinal
with the time scale. The intersection of this asymptote with the vertical scale determines the
coefficient $f_a = (E-D)$ and thus the fractal dimension $D$ that characterizes the specimen set-
up (heat source, specimen, distribution of the temperature field, heat losses). When the value
$f_a$ is known it is feasible to determine the parameters of the studied thermal system with the
aid of the (4) – (9) equations.
Fig. 2 Time dependency of the temperature response for the Dirac thermal pulse (for the heat flow geometry from Fig. 1 calculated by Eq. (2)).

Fig. 3 Time dependency of the reconstructed fractal dimension D for the Dirac thermal pulse and the heat flow geometry from Fig. 1 calculated using Eq (7).
The source type reconstruction (its fractal dimension $D$, coefficient $fa$ respectively) can be determined by Eq. (7). Calculated time dependences of these fractal dimensions for all four cases are shown in Fig. 3. One can see that the results are in agreement with the input parameters. Discrepancies due numerical errors in calculation using Eq. (7) are only for $t \approx t_m$.

### 3. Experimental

For measuring of the responses to the pulse heat the Thermophysical Transient Tester 1.02 was used. It was developed at the Institute of Physics, Slovak Academy of Science [7]. The setup of the experiment is described in [1].

Thermal responses from Slovak Academy were used for the data evaluation. The measured sample of borosilicate wool glass fiber (composite material) was round shaped with diameter $R = 0.03$ m. Its density was $\rho = 77.9$ kg.m$^{-3}$ for its thickness $h = 7.5$ mm, the table thermal conductivity of the matter was $\lambda = 0.0254$ W.m$^{-1}$.K$^{-1}$.

![Diagram of current flow geometry](image)

**Fig. 4** Current flow geometry: a) plane-parallel, b) fractal, c) point (for different ratio of length contact respectively) source.

Three possible configurations of experiment arrangements are shown in Fig. 4. The Fig. 4a shows situation where the diameter of specimen is equal to the diameter of the heat source.
On the other hand, Fig. 4c depicts experimental setup with the diameter of the heat source far smaller than specimen’s diameter. Fig. 4b shows the real situation, when the heat is delivered irregularly (either from the source of finite size (capacity) or from a source with specific composition of heat sources).

4. Results

Typical time responses of temperature for the pulse of input power represent Fig. 5. The width of the heat pulse was $\Delta t = 4\,s$ (not fulfilled symbols, thin lines) and $\Delta t = 100\,ms$ (fulfilled symbol, thick line). It is evident that the position of dependence’s maximum changes with decreasing energy of the pulse to the shorter time. The limit time of these dependences is given by Dirac pulse (experimentally short time pulse $\Delta t = 100\,ms$).

![Graph showing thermal responses](image)

*Fig. 5 The thermal responses of the sample measured by the pulse transient method. Not fulfilled symbols are for pulse width $\Delta t = 4\,s$ and fulfilled symbols stand for pulse width $\Delta t = 100\,ms$.*

The shift of the maximum time and adequate maximal temperature on the power of pulse for different heat power is presented in Fig. 6. Not fulfilled symbols represent thicker pulses and
fulfilled symbols thinner pulses. It is evident (in opposite to ideal model case) that the
maximums are shifted to higher values with increased power of the heat source. Differences
between model and real sample characteristics are probably caused by the heat losses and by
the different properties of composite materials opposite to the homogenous (model) material.

Fig. 6 The dependencies of the position of maximums and adequate temperatures on the heat power.
Not fulfilled symbols are for pulse width \( \Delta t = 4 \) s and fulfilled symbols for pulse width \( \Delta t = 100 \) ms.

The coefficient \( f_a \) (fractal dimension \( D \) respectively) of the fractal heat source for every point
of the experimental dependency (measured temperature on time) was calculated using Eq. (7).
The fractal heat source characterizes the distribution of the temperature in the specimen in
specific time.

It is evident that for very short time the fractal dimension \( D \approx 3 \) and therefore, the volume
heat source is formed. The value of the fractal dimension decreases exponentially with
increasing time since the heat disperses into the space. Time constants \( \tau \) are presented in
Table I. The spatial distribution of the temperature in the sample has not been changed yet in
this area. From the value of the source’s fractal dimension \((D = 2)\) it is possible to determine the coefficient of the heat source \(f_{\omega 0} = 1\) and the thermal diffusivity, thermal capacity and conductivity of the specimen (see Table I). These values are identical to values determined by the Institute of Physic, Slovak Academy of Sciences, Bratislava.

![Graph showing the time dependencies of the fractal dimension \(D\) calculated using Eq. (7) for different power of heat source and different types of pulses.](image)

**Fig. 7** The time dependencies of the fractal dimension \(D\) calculated using Eq. (7) for different power of heat source and different types of pulses.

The deviations between the experimental (Fig. 7) and the model (Fig. 3) responses obvious in the descending part of the characteristics are caused by the heat dissipation from the material via the cylinder surface of the specimen. This causes a faster decrease of the temperature than the theory predicted.

The basic parameters calculated from dependencies in Fig. 5 are presented in Table I. The values of heat energy from the area unit of source to the sample are in the first column. Results for pulses of \(\Delta t = 4s\) width and different power are in the first fourth rows, at the last row, the \(\Delta t = 100\) ms pulse width is presented. At the second and third columns, time and
temperature of maximum dependences in Fig. 5 are listed. Time constants calculated from exponential dependences of fractal dimension on the heat source energy are given in the fourth column. The last three columns give basic values which characterize the thermal properties of materials: thermal diffusivity $a$, thermal capacity $c_p$ and thermal conductivity $\lambda$.

Table I Measured and calculated parameters from measured characteristics

<table>
<thead>
<tr>
<th>$Q/S$ (J.m$^{-2}$)</th>
<th>$t_m$ (s)</th>
<th>$\Delta T_m$ (K)</th>
<th>$\tau$ (s)</th>
<th>$a$ (m$^2$.s$^{-1}$)</th>
<th>$c_p$ (J.kg$^{-1}$.K$^{-1}$)</th>
<th>$\lambda$ (W.m$^{-1}$.K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>228.35</td>
<td>36.05</td>
<td>0.09</td>
<td>29.24</td>
<td>8.44x10$^{-7}$</td>
<td>958.70</td>
<td>0.063</td>
</tr>
<tr>
<td>2038.44</td>
<td>40.16</td>
<td>1.00</td>
<td>23.26</td>
<td>7.57x10$^{-7}$</td>
<td>813.28</td>
<td>0.048</td>
</tr>
<tr>
<td>6156.02</td>
<td>44.06</td>
<td>3.13</td>
<td>18.45</td>
<td>6.90x10$^{-7}$</td>
<td>784.47</td>
<td>0.042</td>
</tr>
<tr>
<td>7012.75</td>
<td>48.43</td>
<td>4.08</td>
<td>12.45</td>
<td>6.28x10$^{-7}$</td>
<td>684.68</td>
<td>0.034</td>
</tr>
<tr>
<td>402.76</td>
<td>36.16</td>
<td>0.13</td>
<td>26.25</td>
<td>8.41x10$^{-7}$</td>
<td>1193.83</td>
<td>0.078</td>
</tr>
</tbody>
</table>

It is evident from these results that the thermal parameters slowly decrease with increasing of heat energy. These variations are caused by increasing heat losses with increasing of pulse power. The dependence is slower for narrow (Dirac) pulses; the typical value is in the last row where the losses are minimal.

5. Conclusion

In this article, the results of thermal responses to the pulse of supplied heat evaluations are discussed. To interpret the outcomes, simplified heat conductivity model is used [1]. The model is based on expectations published in [4]. Results show the image of heat distribution in a specimen in various time intervals after the exposure to the heat from source. These evaluations could be used for more accurate determination of the thermal parameters of studied materials.

Acknowledgments

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References


[8] Thermophysical Transient Tester – Model RT 1.02. Institute of Physic, Slovak Academy of Sciences